第一屆校際系統建模與優化競賽題目

香港某個機構舉辦團體越野賽。按人數分組進行比賽，參賽隊伍帶着自己的隊旗，沿着指定的路綫前進，目標是將自己的隊旗插到最遠的地方。每名參賽隊員除了帶備各自必須的越野裝備外，最重要的是帶備飲用水，而每名參賽隊員有不同的最大負水能力。比賽期間，沒有飲用水補充，但是隊員之間可以共享。當隊中飲用水全部耗盡，該隊便要馬上停止前進，並把隊旗插在當地。任何隊員可以在任何時候退出比賽，並且將他 / 她的飲用水留給其他隊員使用。退出之隊員立即有交通工具接走，因此不會額外消耗飲用水。

問題一

由於比賽期間運動量大，隊員事先不能確定自己行進時的耗水速度。一般來說，身體強壯的隊員，負重能力強，行進的時候，耗水速度也大。為此，組織者隨機挑選了 10 人，事先在賽道上作試驗，以備參賽隊伍參考。試驗結果如下：

最大負水量（公升）	5	6	4.5	5.5	4	7	6.5	8	5	6
平均耗水速度（公升/公里）	0.6	0.6	0.5	0.6	0.5	0.6	0.55	0.7	0.55	0.65

試找出最大負水量和耗水速度之間的大致關係。

問題二

你有一班同學好友，各自的負重能力相差無幾（差別小到可以忽略），計劃組隊參加比賽。請問在雙人組和三人組中，隊員應該如何合作，以把你們的隊旗插到最遠的地方？另外，若為了把隊旗插到一人組所能到達的三倍距離以外，你們最少需要多少隊員合作？

問題三

另外，學校田徑隊請你從隊中 10 名同學裏挑選隊員組成一隊參賽。經過測試，10 名同學的最大負水量是

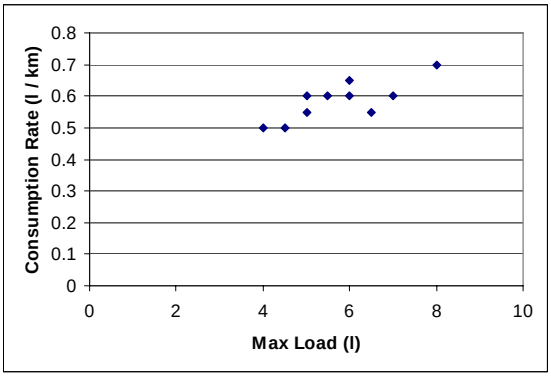
田徑隊員編號	1	2	3	4	5	6	7	8	9	10
負水量（公升）	5	5	5.5	5.5	6	6	6	6	7	7.5

在只參加雙人組 / 三人組的情況下，請分別建議如何挑選同學參賽？如何安排比賽中的合作？行進的最長距離分別是多少？

請就以上三個問題，撰寫一份報告，闡述閣下的見解。

解答一

線性回歸



解答二

假設每名隊員負水 w 公斤，耗水速度 u 公斤/公里。

(a) 記隊員 A 在行走 x 公里后離開，並將水盡量送給 B 隊員。此時 B 隊員一共有水 2w – 2ux 碗. (而且在 2w – 2ux > w 之前，他們是不會如此操作的。當然 x < w/u)。那麼 B 同學還可以行走 (2w – 2ux)/u = 2w/u – 2x 公里，所以隊旗可以插在 x + 2w/u – 2x = 2w/u – x 公里處。于是問題就可以用以下模型來描述：

$$\begin{aligned} &\text{Max} && 2w/u - x \\ &\text{Subject to} && 2w - 2ux \le w \\ & && x \le w/u \end{aligned}$$

用綫性規劃可以得到最優解為：$x = w/(2u)$，最優目標值為 $1.5w/u$ 公里

(b) 記 A 隊員在行走 x 公里後離開，將水平分給 B 和 C。B 和 C 又行走 y 公里後 B 離開，將水留給 C。模型為：

$$\begin{aligned} &\text{Max} && 3w/u - 2x - y \\ &\text{Subject to} && 0 \le 3w - 3xu \le 2w \\ & && 0 \le 3w - 3xu - 2uy \le w \end{aligned}$$

用綫性規劃可以得到最優解為：$x = w/(3u)$, $y = w/(2u)$，最優目標值為 $11w/(6u)$

(c) 通過上述求解，可以猜測在 n 人隊伍中，第一位隊員應該在 $w/(nu)$ 公里處離開，使得剩下的 n-1 名隊員負水 $(n-1)w$。 這樣，n 人隊伍應該比 n-1 人隊伍多走 $w/(n \times u)$ 公里，所以 n 人隊伍可以走 $(1 + 1/2 + 1/3 + \ldots + 1/n) \times (w/u)$。要使得 $1 + \ldots. + 1/n \ge 3$，n 最少需要 11。($1 + ... + 1/10 = 2.929$, $1 + \ldots + 1/11 = 3.02$)。

解答三

(a) 顯然，選 w_i/u_i 最大的同學參賽。根據前面的線性回歸，負水量約大的同學，這個比例越大。所以，選 w_i 最大的同學參賽。

(b) 選 w_i 最大的兩名同學參賽。可以分兩種情況討論：(i) w_i 較大的同學先離開，用前面的線性規劃決定最后的距離。(ii) w_i 較小的同學先離開，用前面的線性規劃決定最后的距離。最后比較哪個距離最大。

(c) 選 w_i 最大的三名同學參賽。分兩步討論第一步決定誰先離開，那麼他離開的時間必定是他/她的水剛好可以使得另外兩人滿負荷帶水，第二步就利用(b)的結論，決定還能行進多遠。這樣，這一問需要比較三中可能。

The 1st Interschool Competition on System Modeling & Optimization (COSMO'06)

A cross-country hiking competition will be held. The competition is divided into several categories according to the team size (e.g., single-person, two-person, three-person, etc.). Every team will carry a team flag and travel along a defined route. The goal is to carry the flag and travel the longest distance from the starting point. The farthest team will win the competition. Every team member will not only bring along the necessary cross-country hiking equipment, but also the necessary drinking water. Every member may have different water carrying capability. During the course of the competition, there will not be any supply of additional drinking water. However, members from the same team can share drinking water. When all drinking water in a team is exhausted, the team must stop going further and the traveled distance is measured up to that point. In addition, a team can arrange any member to leave the team at any time and transfer his/her remaining drinking water to other team members still in the team. The organizer will pick up the left member and hence he/she will not consume drinking water of the team anymore.

Question 1:

Due to the toughness of the competition, participants cannot pre-estimate the exact drinking water consumption during the course of competition. However, it is generally known that a strong member can carry more drinking water but the water consumption rate is also higher. In order to get a better understanding of this, the organizer has conducted a test by randomly choosing 10 people to try the hiking route and make measurement of their water carrying capacity and drinking water consumption rate for reference of the participants. The test results are listed as follows.

Max Water Carrying Capacity（L）	5	6	4.5	5.5	4	7	6.5	8	5	6
Avg. Water Consumption Rate（L/km）	0.6	0.6	0.5	0.6	0.5	0.6	0.55	0.7	0.55	0.65

Please try to find out a rough relationship between maximum water carrying capacity and water consumption rate.

Question 2:

You have a group of friends each with approximately the same water carrying capacity (the difference can be assumed to be negligible) and you are planning to form a team to participate in the competition. When you form the team, how should the members cooperate in drinking water arrangement in order to travel the farthest for the team in the two-person category and the three-person category? If we want to carry the flag a distance three-time that of a single-person team can travel, at least how many members are needed in such team?

Question 3:

The athletic team of your school asks you to form a team from the 10 members to participate this cross-country hiking competition. After some tests, the measured maximum water carrying capacities of the 10 athletic team members are given below.

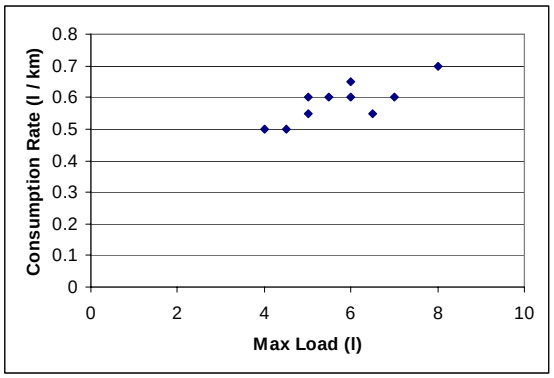
Athletic Team Member No.	1	2	3	4	5	6	7	8	9	10
Water Carrying Capacity (liter)	5	5	5.5	5.5	6	6	6	6	7	7.5

In the two-person category and three-person category respectively, please recommend how to select members and arrange their cooperation in the competition in order to travel the farthest.

Please write a report to elaborate your recommendations.

Answer 1

Using linear regression:



Answer 2

Assume the maximum water carrying capacity of each participant is w liter and the average water consumption rate is u liter/km.

(a) After member A has traveled x km, he leaves and transfers his remaining water to member B. Now, member B has 2w – 2ux liter drinking water. (They will not do it when 2w – 2ux > w, and certainly x < w/u). In this way, member B can travel (2w – 2ux)/u = 2w/u – 2x km more, and thus the team flag can reach x + 2w/u – 2x = 2w/u – x km. We can formulate the following model:

Max 2w/u – x
Subject to 2w – 2ux ≤ w
x ≤ w/u

By linear regression, the optimal solution we get is: x = w/(2u), with 1.5w/u km.

(b) After member A has traveled x km, he leaves and equally transfers his water to members B and C. B and C then continue to travel y km, after that, B leaves and transfers all his water to C. The model can be formulated as follow:

Max 3w/u – 2x – y
Subject to 0 ≤ 3w – 3xu ≤ 2w
0 ≤ 3w – 3xu – 2uy ≤ w

By linear regression, optimal solution: x = w/(3u), y = w/(2u), and the optimal value: 11w/(6u).

(c) From the above solutions, we can infer that in a team of n members, the first member should leave at w/(n×u) km, the remaining (n-1) members will then carry (n-1)w liter water. In this way, a team with n members should be able to travel w/(n×u) km more than a team with (n-1) members, therefore a n-member team can travel (1 + 1/2 + 1/3 +…+ 1/n)w/u. To make 1 +…+ 1/n ≥ 3, n should at least be 11. (1 + … + 1/10 = 2.929, 1 + … + 1/11 = 3.02)

Answer 3

(a) Obviously, we should recommend studetents with the largest w_i/u_i to participate this competition. According to the linear regression in question 1, students with larger water carrying capacity would have larger w_i/u_i ratio. Thus, we should recommend students with the largest w_i to participate in the competition.

(b) We should recommend two students with the largest w_i. This recommendation would lead to two scenarios: (i) The student with larger w_i leaves first, then determine the distance trvael by the above linear programming. (ii) The student with smaller w_i leaves first, then derermine the distance trvael by the above linear programming. Choose the scenario with the longer distance travel.

(c) We should recommend three students with the largest w_i. Te deal with this problem, we need to divide the problem into 2 steps. In the first step, determine which student should leave first, so that at the time when he leaves, his remaining water is equal to the sum of the maximum water carrying capacity of the other two students. In the second step, using the result of (b), determine the distance travel. Therefore, we need to consider three scenarios of this problem.