

# COSMO 2022 學生講座

香港中文大學系統工程及工程管理學系



# 學生講座

- 上午：數學建模簡介及例子
  - 基本數學建模概念
  - 建立系統模型並作優化
- 下午：優化模型實戰 (Microsoft Excel)
  - 運用數學方法與計算技術



# 下午講座

- Microsoft Excel Solver 基本操作簡介
- Microsoft Excel 實際應用例題-線性迴歸



# Microsoft Excel Solver 基本操作簡介

- 我們考慮以下線性規劃 (Linear Programming, LP) 。並以此用作 Microsoft Excel Solver 基本操作簡介。

$$\text{Max} \quad x + y$$

*s. t.*

$$1.5x + y \leq 9$$

$$150x + 70y \leq 750$$

$$y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$

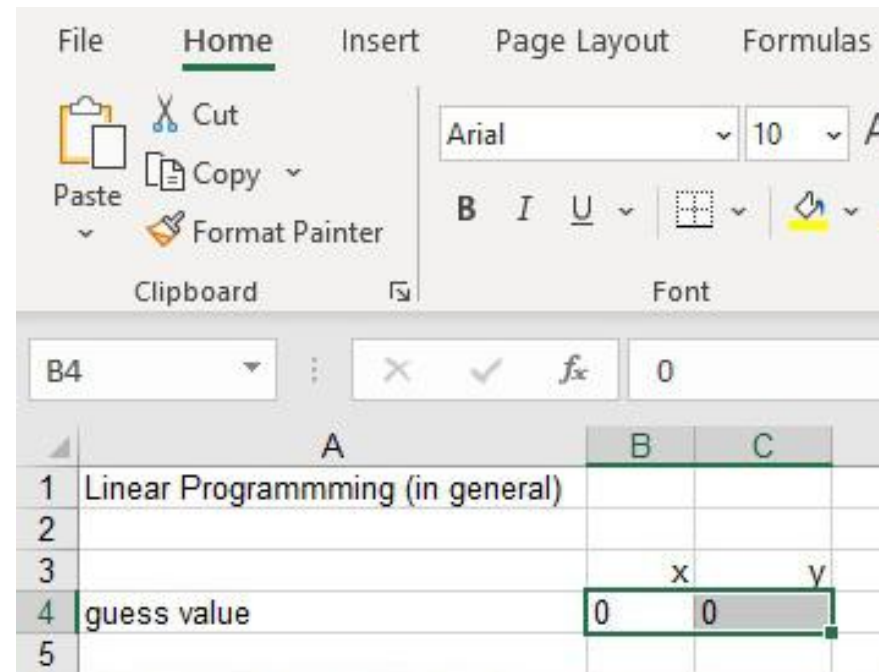
$$\text{Max } x + y$$

s. t.

$$\begin{aligned} 1.5x + y &\leq 9 \\ 150x + 70y &\leq 750 \\ y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

# Microsoft Excel Solver 基本操作簡介

- 首先我們需要自訂及確立變數 (variable) 格位置。如圖中，B4 及 C4 分別選定為變數 x 和 y 的變數格。



Max

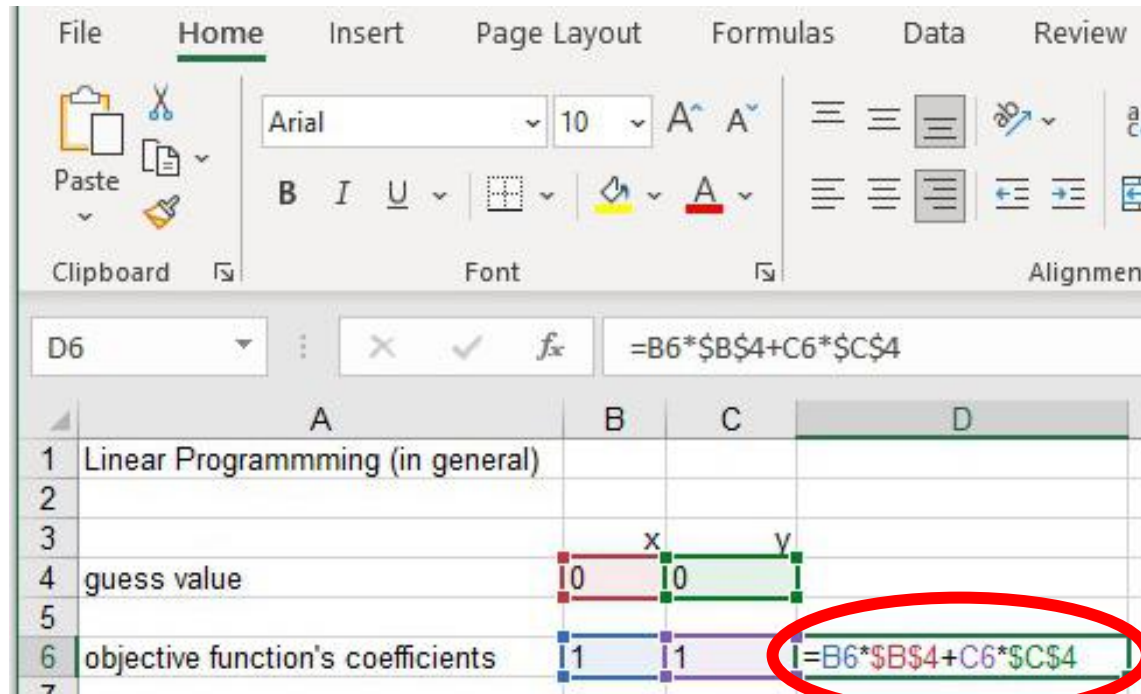
$x + y$

s. t.

$$\begin{aligned} 1.5x + y &\leq 9 \\ 150x + 70y &\leq 750 \\ y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

# Microsoft Excel Solver 基本操作簡介

- 輸入目標函數 (objective function) 的系數 (coefficients) 。並訂下目標函數格 D6 。



← 目標函數的公式

Max

$x + y$

s. t.

$$\begin{aligned} 1.5x + y &\leq 9 \\ 150x + 70y &\leq 750 \\ y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

# Microsoft Excel Solver 基本操作簡介

- 輸入有關約束 (constraint) 的系數，並訂下各約束的數學關係 (即是各約束的「左邊」)。  
(暫時可以先忽略非負數的條件。)

	A	B	C	D	E	F
1	Linear Programmimg (in general)					
2						
3						
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4		
7						
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6
11						

各約束的「左邊」

# Microsoft Excel Solver 基本操作簡介

- 當完成上述設定。我們點選目標函數格，然後在“Data”選取“Solver”。

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Linear Programming (in general)														
2															
3		x	y												
4	guess value	0	0												
5															
6	objective function's coefficients	1		=B6*\$B\$4+C6*\$C\$4											
7															
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9									
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750									
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6									
11															



# Microsoft Excel Solver 基本操作簡介

- 按需要選擇“Max”或”Min”，在“By Changing Variable Cells”選定變數格。

The screenshot displays the Microsoft Excel Solver interface. The Solver Parameters dialog box is open, showing the following configuration:

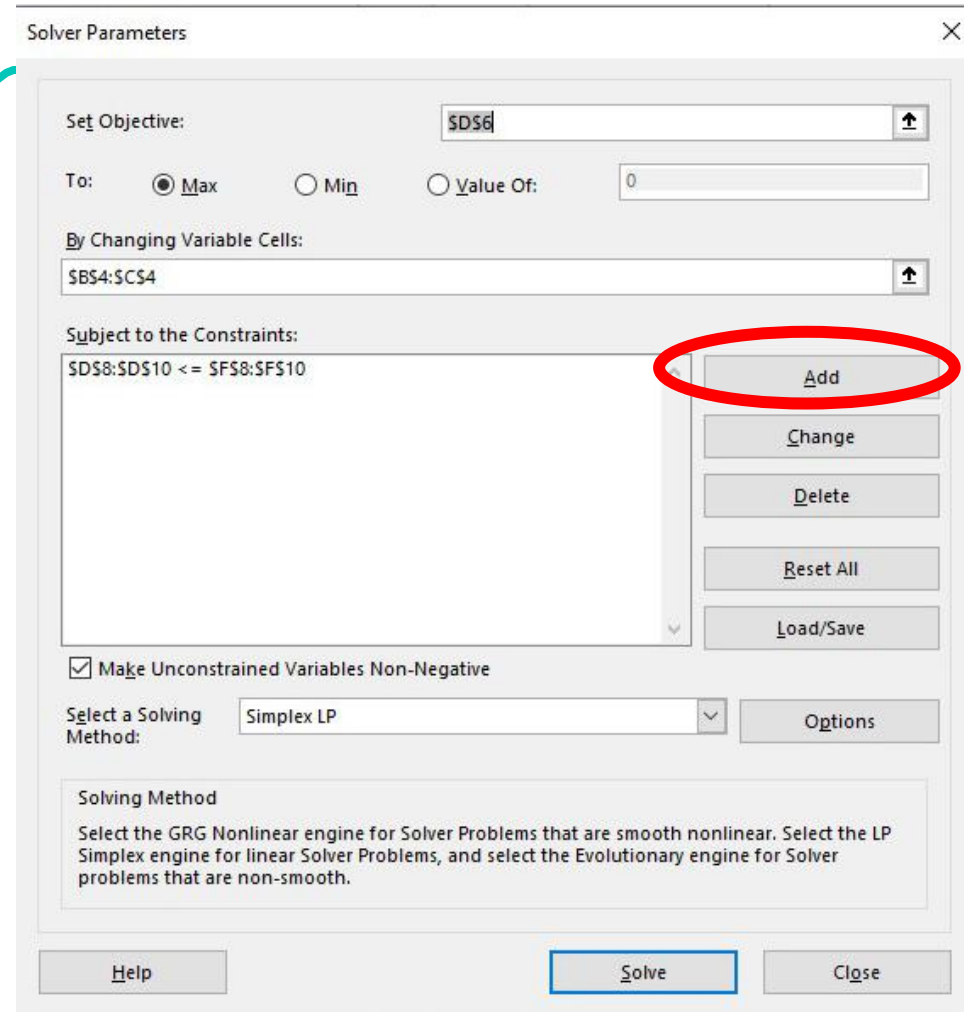
- Set Objective:**  $\$D\$6$
- To:**  Max  Min  Value Of: 0
- By Changing Variable Cells:**  $\$B\$4:\$C\$4$

The background spreadsheet shows a linear programming problem setup:

	A	B	C	D	E	F
1	Linear Programming (in general)					
2						
3		x	y			
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	$=B6*\$B\$4+C6*\$C\$4$		
7						
8	constraints	1.5	1	$=B8*\$B\$4+C8*\$C\$4$	$\leq 9$	
9		150	70	$=B9*\$B\$4+C9*\$C\$4$	$\leq 750$	
10		0	1	$=B10*\$B\$4+C10*\$C\$4$	$\leq 6$	

# Microsoft Excel Solver 基本操作簡介

- 在Solver上按“Add”加入約束。
- 之後會彈出一個新視窗。



# Microsoft Excel Solver 基本操作簡介

- 在“Call Reference”選取剛才定下的“左邊”式
- “Constraint”格就是對應的“右邊”。
- 按OK。

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F
1	Linear Programming (in general)					
2						
3		x	y			
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4		
7						
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						

The 'Add Constraint' dialog box is open, showing the following fields:

- Cell Reference: \$D\$8
- Constraint: =\$F\$8
- Operator: <=

Green arrows point from the dialog box fields to the corresponding cells in the spreadsheet: from 'Cell Reference' to D8, from 'Constraint' to F8, and from the operator to the cell between D8 and F8.

# Microsoft Excel Solver 基本操作簡介

- 如是者，加入所有約束。

	A	B	C	D	E	F	G
1	Linear Programming (in general)						
2							
3		x	y				
4	guess value	0	0				
5							
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4			
7							
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4		<= 9	
9		150	70	=B9*\$B\$4+C9*\$C\$4		<= 750	
10		0	1	=B10*\$B\$4+C10*\$C\$4		<= 6	
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							

Solver Parameters

Set Objective:

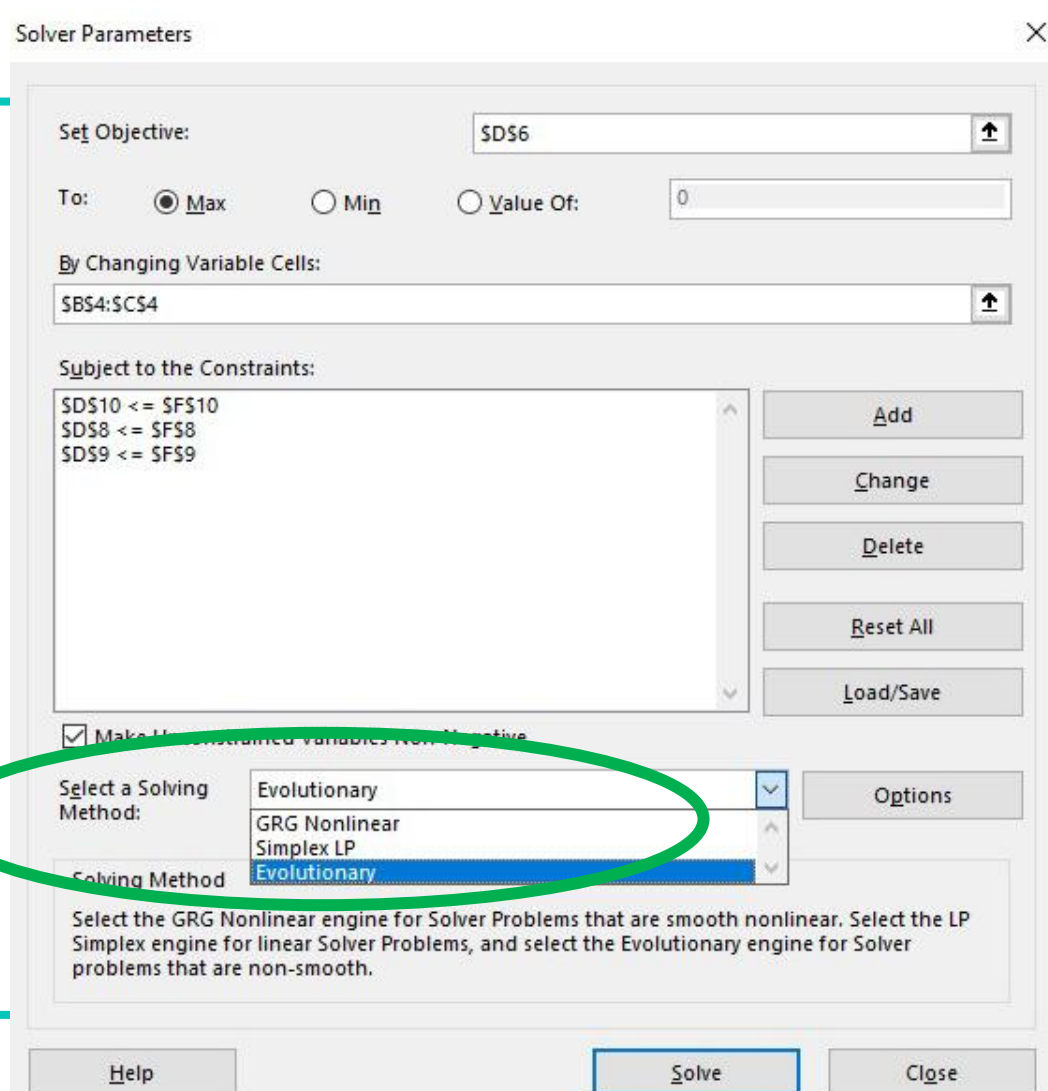
To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

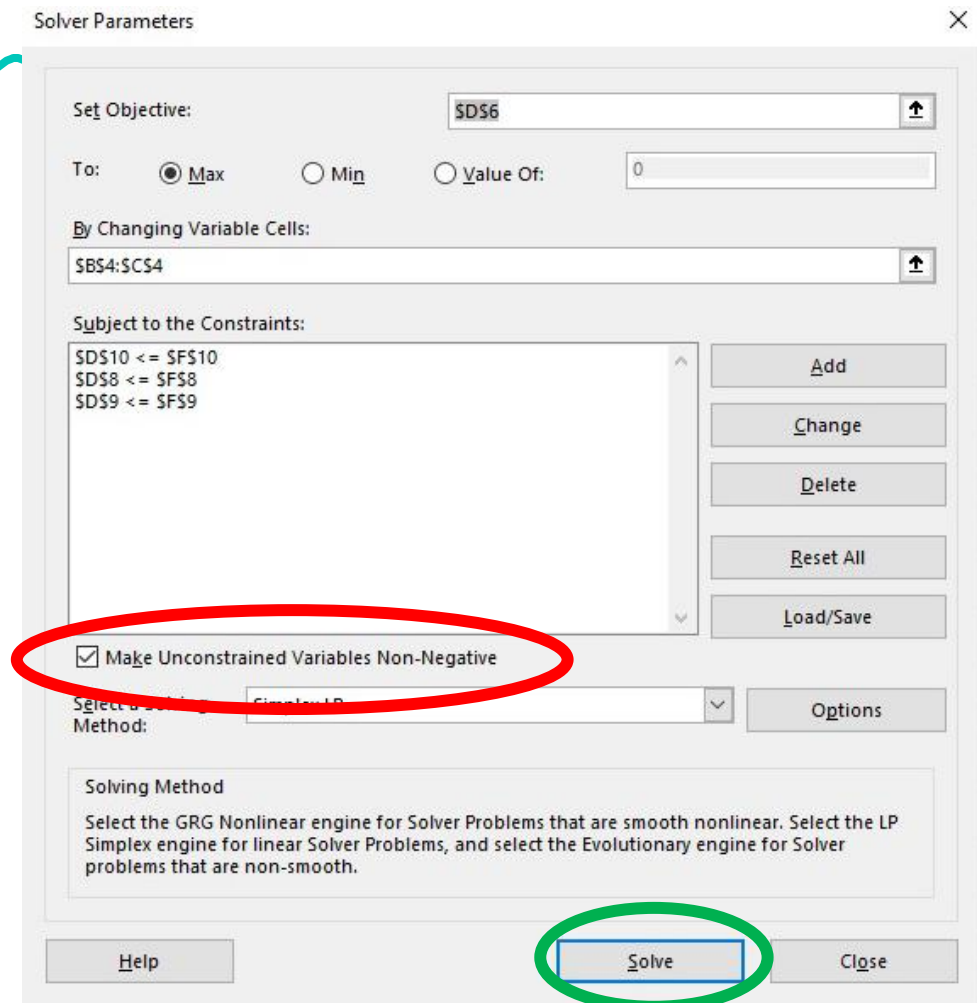
# Microsoft Excel Solver 基本操作簡介

- 在” **Select a Solving Method**” 中，我們可以選擇不同解決方法：
- “**Simplex LP**” 主要應用在線性問題上。
- “**GRG Nonlinear**” 一般應用到非線性問題上。
- “**Evolutionary**” 一般能夠應用到更多不同優化的問題。



# Microsoft Excel Solver 基本操作簡介

- 一般情況下，Solver 已預先設定或加入非負數的條件。
- 完成後，按“Solve”，便能求出答案。



# Microsoft Excel Solver 基本操作簡介

- 可按“Keep Solver Solution”保留答案。
- 在 D6 顯示出由Solver 給出的答案。

The screenshot displays an Excel spreadsheet with a linear programming problem set up in columns A through J. The problem is titled "Linear Programming (in general)". The objective function is in cell D6, with a value of 8. The constraints are listed in rows 8, 9, and 10. The Solver Results dialog box is open, showing that a solution has been found and that all constraints and optimality conditions are satisfied. The "Keep Solver Solution" option is selected and circled in blue. The "Restore Original Values" option is unselected. The "Return to Solver Parameters Dialog" and "Outline Reports" options are also unselected. The "OK" button is highlighted with a dashed border.

	A	B	C	D	E	F	G	H	I	J
1	Linear Programming (in general)									
2										
3		x	y							
4	guess value	2	6							
5										
6	objective function	1	1	8						
7										
8	constraints	1.5	1	9	<=	9				
9		150	70	720	<=	750				
10		0	1	6	<=	6				
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22										
23										
24										
25										
26										
27										

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

Keep Solver Solution

Restore Original Values

Return to Solver Parameters Dialog

Outline Reports

Reports

Answer

Sensitivity

Limits

OK Cancel Save Scenario...

# Microsoft Excel Solver 基本操作簡介

- 當有需要將變量設定為整數變量或者二進制變量時，我們可以在設定約束時選取有關變量為
  - 整數變量(int); 或
  - 二進制變量 (bin)

Linear Programming (in general)

	x	y			
guess value	2	6			
objective function'	1	1	8		
constraints	1.5	1	9	<=	9
	150	70	720	<=	750
	0	1	6	<=	6

Add Constraint

Cell Reference: \$B\$4

Constraint: <=

OK Cancel

int  
bin  
dif



# Microsoft Excel Solver 基本操作簡介

我們可以在目標函數中加入一些特別的數學函數。以下讓我們來介紹兩個數學函數。

○ (I)

$$\text{Max}(x_1, x_2) = \begin{cases} x_2, & x_1 \leq x_2 \\ x_1, & x_1 > x_2 \end{cases}$$

○ (II)

$$\text{Min}(x_1, x_2) = \begin{cases} x_1, & x_1 \leq x_2 \\ x_2, & x_1 > x_2 \end{cases}$$

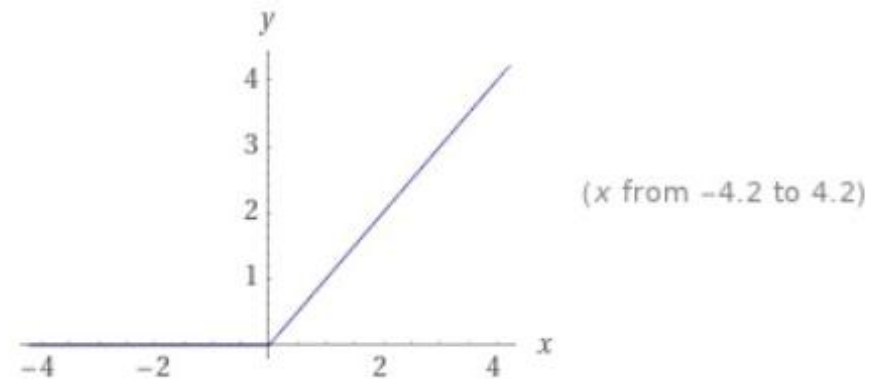
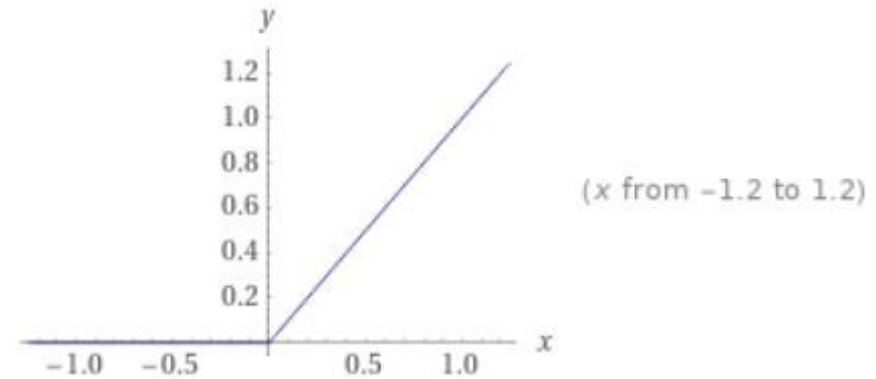
# Microsoft Excel Solver 基本操作簡介

Source: <https://www.wolframalpha.com/input?i=max%280%2C+x%29>

$$\text{Max}(x_1, x_2) = \begin{cases} x_2, & x_1 \leq x_2 \\ x_1, & x_1 > x_2 \end{cases}$$

(I) 例子:

$$\text{Max}(x, 0) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$



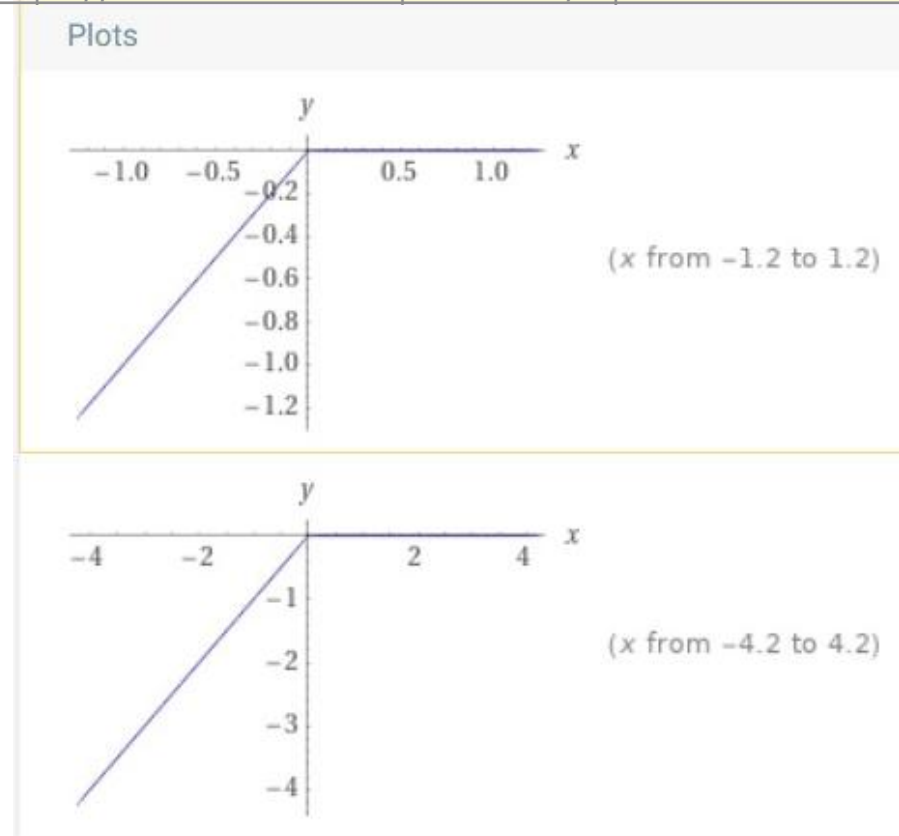
# Microsoft Excel Solver 基本操作簡介

Source: <https://www.wolframalpha.com/input?i=min%280%2C+x%29>

$$\text{Min}(x_1, x_2) = \begin{cases} x_1, & x_1 \leq x_2 \\ x_2, & x_1 > x_2 \end{cases}$$

II. 例子:

$$\text{Min}(x, 0) = \begin{cases} x, & x \leq 0 \\ 0, & x > 0 \end{cases}$$



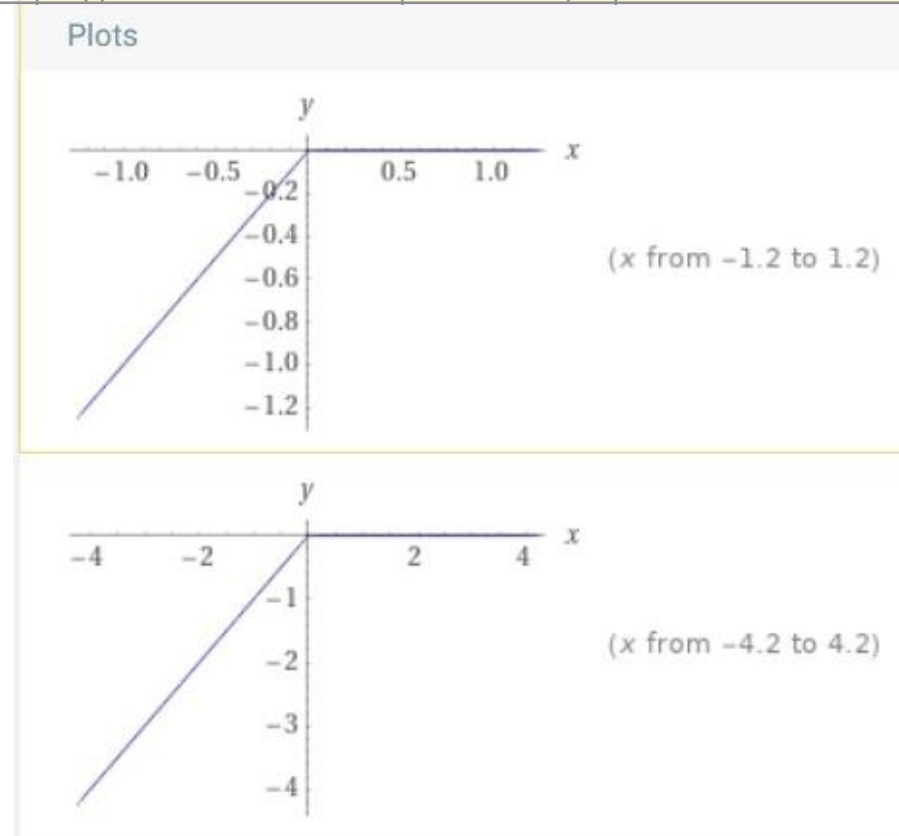
# Microsoft Excel Solver 基本操作簡介

Source: <https://www.wolframalpha.com/input?i=min%280%2C+x%29>

$$\text{Min}(x_1, x_2) = \begin{cases} x_1, & x_1 \leq x_2 \\ x_2, & x_1 > x_2 \end{cases}$$

II. 例子:

$$\text{Min}(x, 0) = \begin{cases} x, & x \leq 0 \\ 0, & x > 0 \end{cases}$$

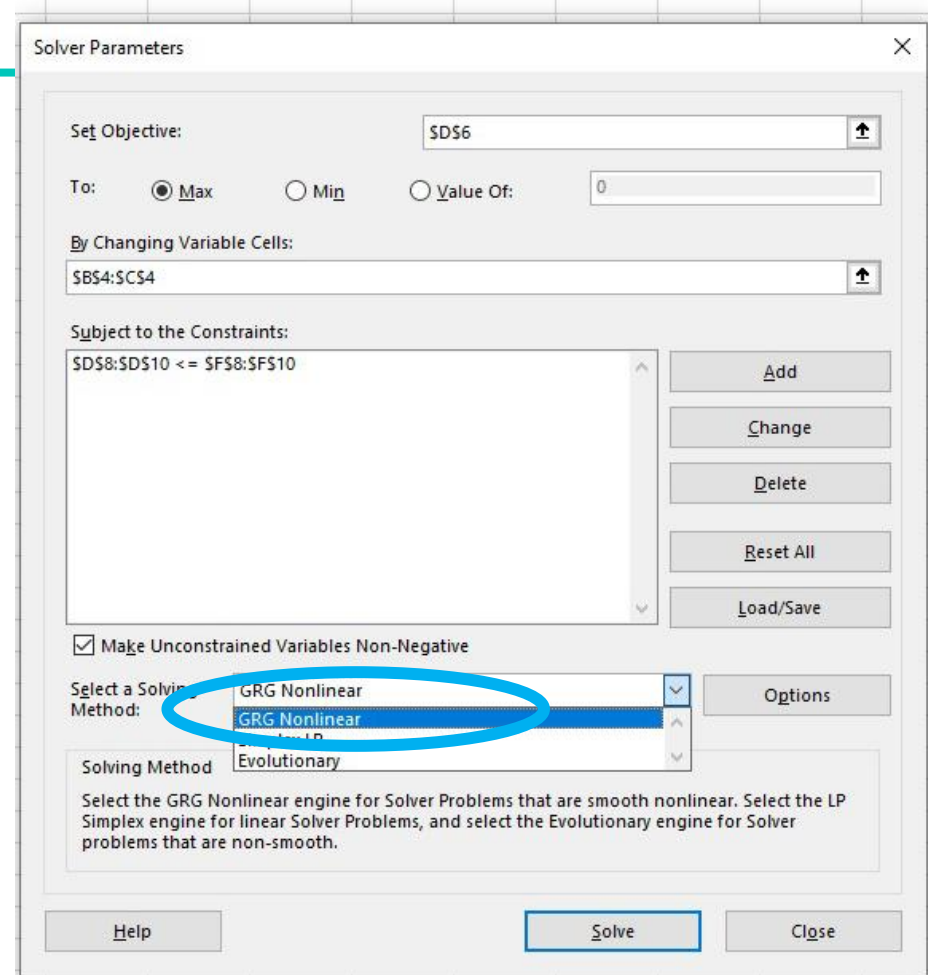


# Microsoft Excel Solver 基本操作簡介

○ 例子：

$$\begin{aligned} &\text{maximize}_{x,y} && x + \min\{2, y\} \\ &\text{subject to} && 1.5x + y \leq 9 \\ & && 150x + 70y \leq 750 \\ & && 0 \leq y \leq 6, \quad 0 \leq x \end{aligned}$$

○ 解決以上問題時，我們選擇“GRG Nonlinear”求解。



# Microsoft Excel的應用-線性迴歸

- 假設一家公司有七名員工。下表列出了上年度員工請假天數的數據資料。

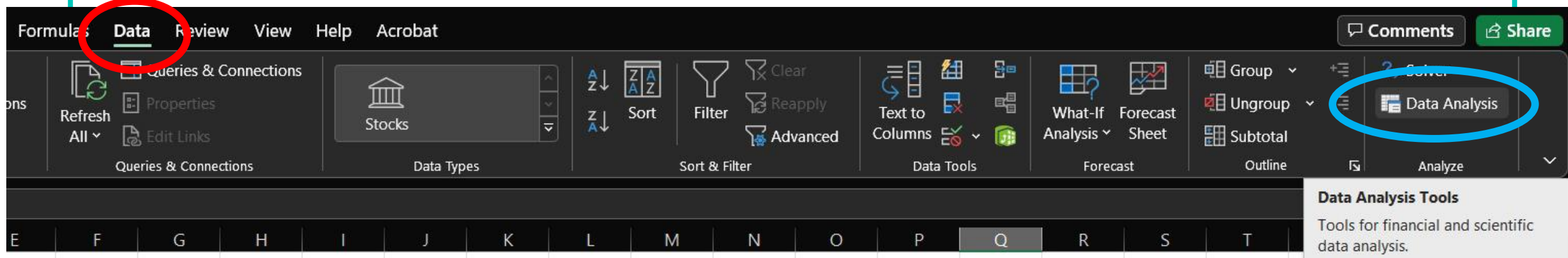
- X: 員工的年資
- Y: 上年度請假天數

X	Y
2	8
5	7
7	5
3	12
8	3
3	9
7	5

假如我們希望使用線性迴歸分析員工年資 (X) 和請假天數 (Y) 的線性關係。

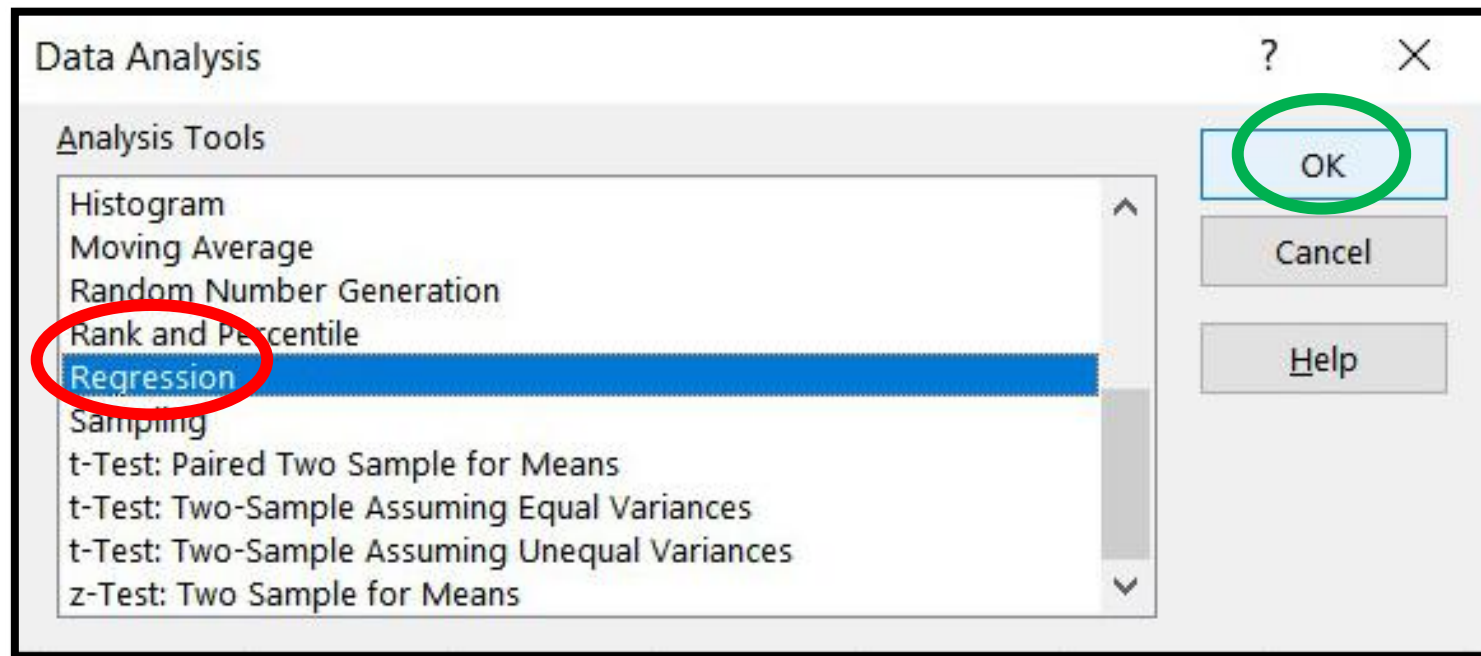
# Microsoft Excel的應用-線性迴歸

- 在“Data”選取“Data Analysis”。接著會彈出一個新的視窗。



# Microsoft Excel的應用-線性迴歸

- 在新視窗中選取“Regression” 並按“OK”。





# Microsoft Excel的應用-線性迴歸

○ 我們可以按圖中示範，在不同位置輸入各項資料及設定。

○ 例如：

○ Y 變量的範圍是 \$B\$1:\$B\$8

○ X 變量的範圍是 \$A\$1:\$A\$8

○ 當設定完畢，按“OK”。

The screenshot displays an Excel spreadsheet with data in columns A and B, and the Regression dialog box open. The data table is as follows:

	A	B
1	X	Y
2	2	8
3	5	7
4	7	5
5	3	12
6	8	3
7	3	9
8	7	5

The Regression dialog box is configured with the following settings:

- Input Y Range: \$B\$1:\$B\$8
- Input X Range: \$A\$1:\$A\$8
- Labels
- Constant is Zero
- Confidence Level: 90 %
- Output options:
  - Output Range: \$A\$10
  - New Worksheet Ply:
  - New Workbook
- Residuals:
  - Residuals
  - Residual Plots
  - Standardized Residuals
  - Line Fit Plots
- Normal Probability:
  - Normal Probability Plots

Buttons: OK, Cancel, Help

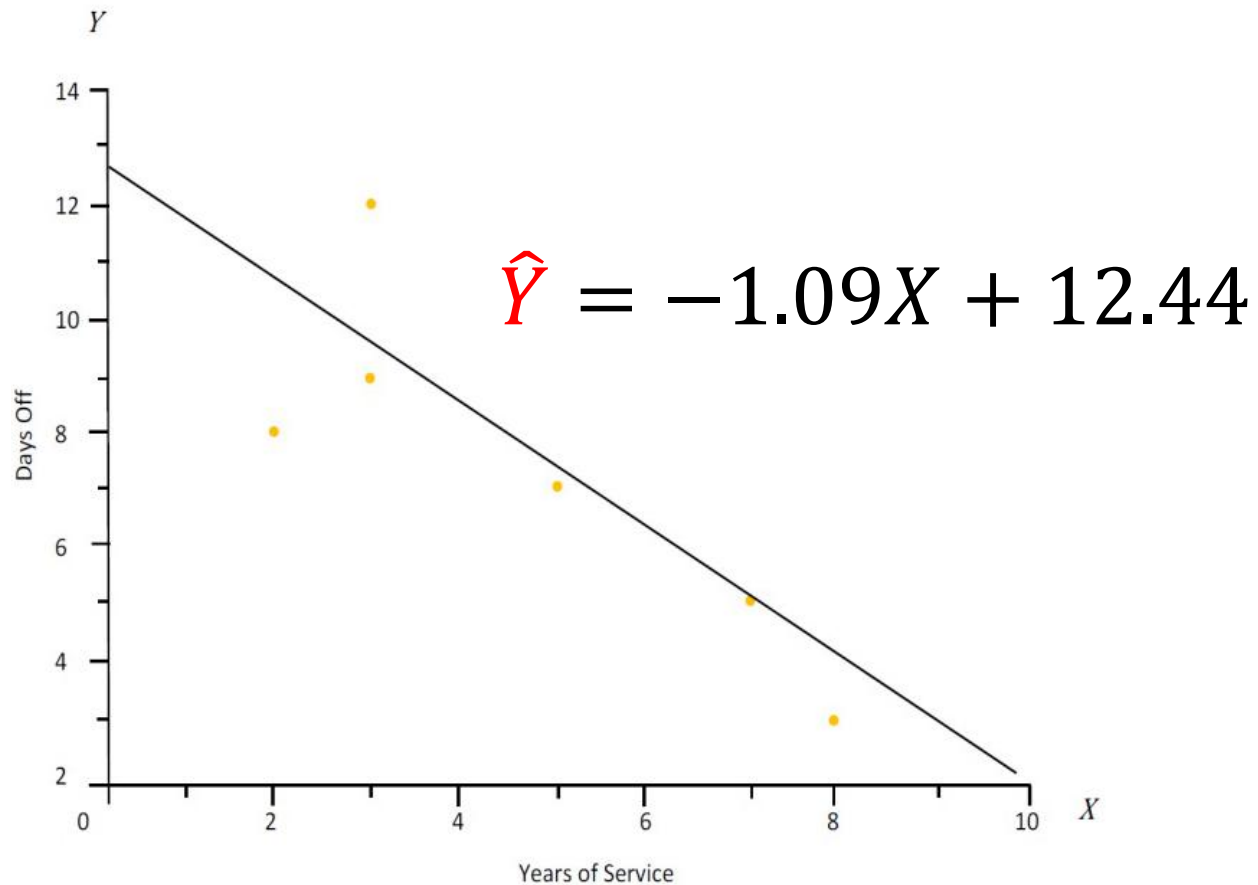
# Microsoft Excel的應用-線性迴歸

Y軸截距  
(Y-intercept)

斜率  
(Slope)

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.863506052							
R Square	0.745642702							
Adjusted R	0.694771242							
Standard E	1.65742536							
Observatic	7							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	40.26470588	40.26470588	14.65738758	0.012266446			
Residual	5	13.73529412	2.747058824					
Total	6	54						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90.0%</i>	<i>Upper 90.0%</i>
Intercept	12.44117647	1.553168751	8.01018979	0.000489973	8.448629092	16.43372385	9.311466306	15.5708866
X	-1.088235294	0.284246104	-3.828496778	0.012266446	-1.818913166	-0.357557422	-1.661004944	-0.51546564

# Microsoft Excel的應用-線性迴歸



# 總結

- Microsoft Excel Solver 基本操作簡介
  - 基本操作簡介
  - 設定整數變量或者二進制變量
  - 特別數學函數介紹: Max 和 Min
- Microsoft Excel的應用-線性迴歸