

COSMO 2024 學生講座

香港中文大學系統工程及工程管理學系



學生講座

- 上午：數學建模簡介及例子
 - 基本數學建模概念
 - 建立系統模型並作優化
- 下午：優化模型實戰 (Microsoft Excel)
 - 運用數學方法與計算技術



下午講座

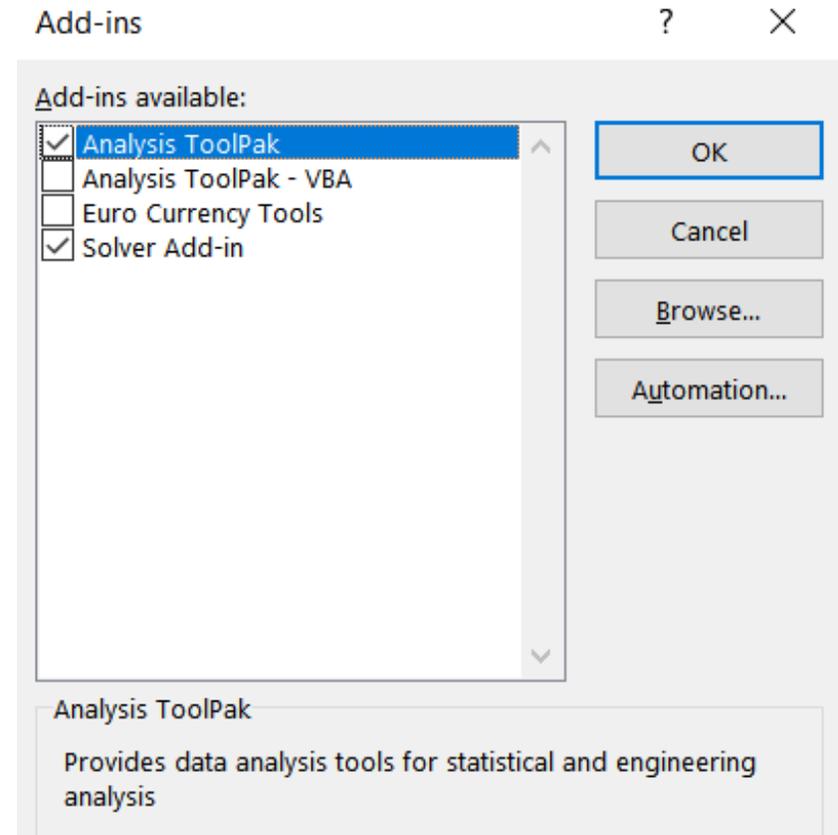
- Microsoft Excel Solver 基本操作簡介
- Microsoft Excel 實際應用例題-線性迴歸



Microsoft Excel Solver 和 Analysis ToolPak 安裝

- Microsoft Excel Solver 安裝流程

- <https://support.microsoft.com/en-us/office/load-the-solver-add-in-in-excel-612926fc-d53b-46b4-872c-e24772f078ca#OfficeVersion=Windows>



Microsoft Excel Solver 基本操作簡介

- 我們考慮以下線性規劃 (Linear Programming, LP) 。並以此用作 Microsoft Excel Solver 基本操作簡介。

$$\text{Max} \quad x + y$$

s. t.

$$1.5x + y \leq 9$$

$$150x + 70y \leq 750$$

$$y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$

Max

$x + y$

s. t.

$$\begin{aligned} 1.5x + y &\leq 9 \\ 150x + 70y &\leq 750 \\ y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Microsoft Excel Solver 基本操作簡介

- 輸入目標函數 (objective function) 的系數 (coefficients) 。並訂下目標函數格 D6 。

The screenshot shows the Microsoft Excel Solver interface. The Solver Parameters dialog box is open, with the objective function cell set to D6. The formula bar shows the formula $=B6*\$B\$4+C6*\$C\4 . The spreadsheet below shows the following data:

	A	B	C	D
1	Linear Programmimg (in general)			
2				
3		x	y	
4	guess value	10	0	
5				
6	objective function's coefficients	1	1	$=B6*\$B\$4+C6*\$C\4
7				

← 目標函數的公式

$$\text{Max } x + y$$

s. t.

$$\begin{aligned} 1.5x + y &\leq 9 \\ 150x + 70y &\leq 750 \\ y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Microsoft Excel Solver 基本操作簡介

- 輸入有關約束 (constraint) 的系數，並訂下各約束的數學關係 (即是各約束的「左邊」)。
(暫時可以先忽略非負數的條件。)

	A	B	C	D	E	F
1	Linear Programming (in general)					
2						
3		x	y			
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4		
7						
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6
11						

各約束的「左邊」

Microsoft Excel Solver 基本操作簡介

- 當完成上述設定。我們點選目標函數格，然後在“Data”選取“Solver”。

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Linear Programming (in general)														
2															
3			x	y											
4	guess value	0	0												
5															
6	objective function's coefficients	1		=B6*\$B\$4+C6*\$C\$4											
7															
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9									
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750									
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6									
11															

Microsoft Excel Solver 基本操作簡介

- 按需要選擇“Max”或”Min”，在“By Changing Variable Cells”選定變數格。

The screenshot shows the Microsoft Excel Solver Parameters dialog box overlaid on a spreadsheet. The spreadsheet is titled "Linear Programming (in general)" and contains the following data:

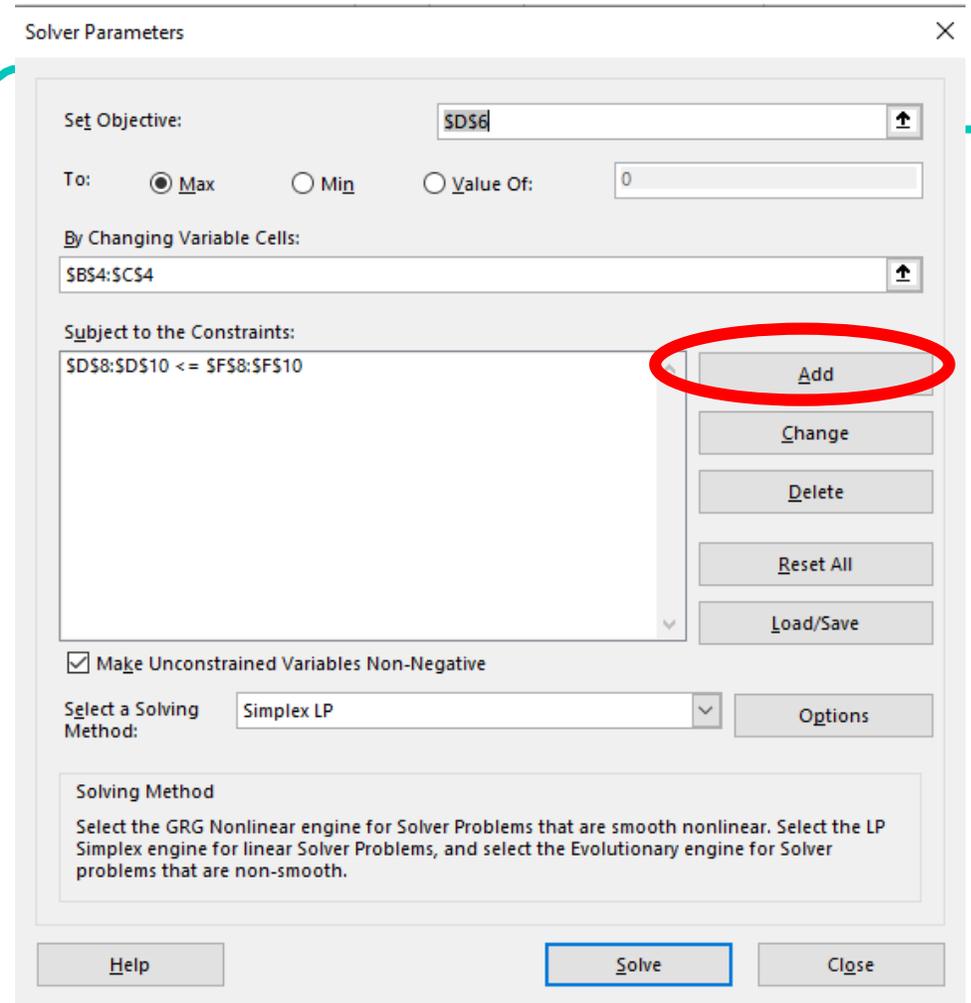
	A	B	C	D	E	F
1	Linear Programming (in general)					
2						
3		x	y			
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4		
7						
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6

The Solver Parameters dialog box is open, showing the following settings:

- Set Objective: \$D\$6
- To: Max Min Value Of: 0
- By Changing Variable Cells: \$B\$4:\$C\$4

Microsoft Excel Solver 基本操作簡介

- 在Solver上按“Add”加入約束。
- 之後會彈出一個新視窗。



Microsoft Excel Solver 基本操作簡介

- 在“Cell Reference”選取剛才定下的“左邊”式
- “Constraint”格就是對應的“右邊”。
- 按OK。

The screenshot displays an Excel spreadsheet with a linear programming problem set up in columns A through F. The problem is titled "Linear Programming (in general)".

	A	B	C	D	E	F
1	Linear Programming (in general)					
2						
3		x	y			
4	guess value	0	0			
5						
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4		
7						
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4	<=	9
9		150	70	=B9*\$B\$4+C9*\$C\$4	<=	750
10		0	1	=B10*\$B\$4+C10*\$C\$4	<=	6
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						

The "Add Constraint" dialog box is open, showing the following fields:

- Cell Reference: \$D\$8
- Constraint: =\$F\$8
- Operator: <=

Green arrows point from the dialog box fields to the corresponding cells in the spreadsheet: from "Cell Reference" to cell D8, from "Constraint" to cell F8, and from the operator "<=" to the operator cell between D8 and F8.

Microsoft Excel Solver 基本操作簡介

- 如是者，加入所有約束。

	A	B	C	D	E	F	G
1	Linear Programming (in general)						
2							
3		x	y				
4	guess value	0	0				
5							
6	objective function's coefficients	1	1	=B6*\$B\$4+C6*\$C\$4			
7							
8	constraints	1.5	1	=B8*\$B\$4+C8*\$C\$4		<= 9	
9		150	70	=B9*\$B\$4+C9*\$C\$4		<= 750	
10		0	1	=B10*\$B\$4+C10*\$C\$4		<= 6	
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							
27							

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

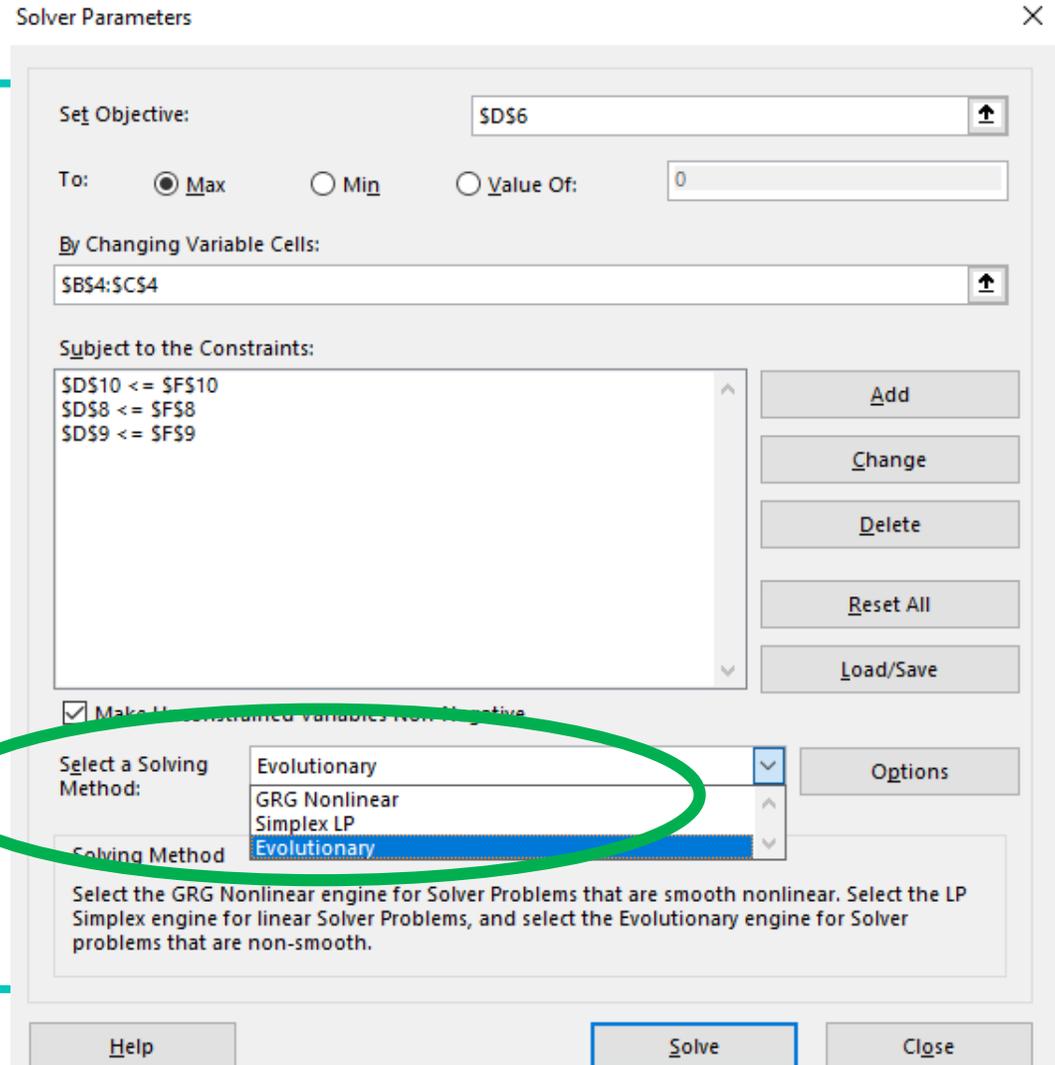
Subject to the Constraints:

- SD\$10 <= SF\$10
- SD\$8 <= SF\$8
- SD\$9 <= SF\$9

Add Change

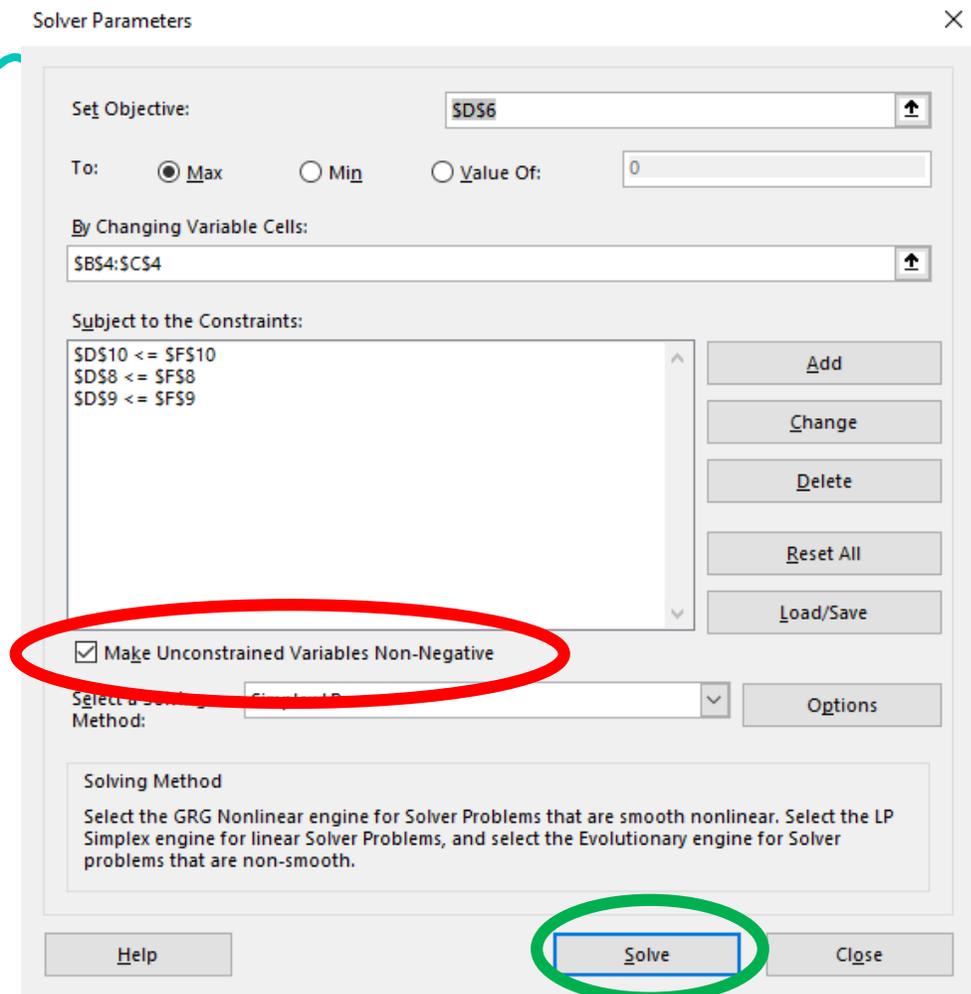
Microsoft Excel Solver 基本操作簡介

- 在” **Select a Solving Method**” 中，我們可以選擇不同解決方法：
- “**Simplex LP**” 主要應用在線性問題上。
- “**GRG Nonlinear**” 一般應用到非線性問題上。
- “**Evolutionary**” 一般能夠應用到更多不同優化的問題。



Microsoft Excel Solver 基本操作簡介

- 一般情況下，Solver 已預先設定或加入非負數的條件。
- 完成後，按“**Solve**”，便能求出答案。



Microsoft Excel Solver 基本操作簡介

- 可按“Keep Solver Solution”保留答案。
- 在 D6 顯示出由Solver 給出的答案。

The screenshot displays an Excel spreadsheet with a linear programming problem and the Solver Results dialog box. The spreadsheet data is as follows:

	A	B	C	D	E	F	G	H	I	J
1	Linear Programming (in general)									
2										
3		x	y							
4	guess value	2	6							
5										
6	objective function'	1	1	8						
7										
8	constraints	1.5	1	9	<=	9				
9		150	70	720	<=	750				
10		0	1	6	<=	6				
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22										
23										
24										
25										
26										
27										

The Solver Results dialog box is open, showing the following options:

- Keep Solver Solution
- Restore Original Values
- Return to Solver Parameters Dialog
- Outline Reports

The "Keep Solver Solution" option is highlighted with a blue oval. The "Reports" section includes "Answer", "Sensitivity", and "Limits". The "OK" button is also highlighted with a blue oval.

Microsoft Excel Solver 基本操作簡介

- 假如我們發現之前討論的線性規劃的目標函數是不正確。。
- 正確的目標函數更新如下，我們仍然可以用 Microsoft Excel Solver 找出最優的解。

$$\text{Max} \quad x^2 + y$$

s. t.

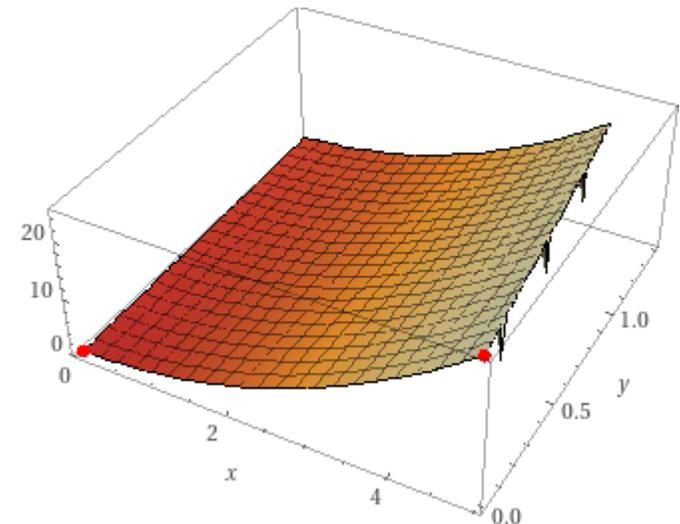
$$1.5x + y \leq 9$$

$$150x + 70y \leq 750$$

$$y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$



Microsoft Excel Solver 基本操作簡介

- 當有需要將變量設定為整數變量或者二進制變量時，我們可以在設定約束時選取有關變量為
 - 整數變量(int); 或
 - 二進制變量 (bin)

Linear Programming (in general)

	x	y			
guess value	2	6			
objective function'	1	1	8		
constraints	1.5	1	9	<=	9
	150	70	720	<=	750
	0	1	6	<=	6

Add Constraint

Cell Reference: \$B\$4

Constraint: <=

OK Cancel

int
bin
dif

Microsoft Excel Solver 基本操作簡介

我們可以在目標函數中加入一些特別的數學函數。以下讓我們來介紹兩個數學函數。

○ (I)

$$\text{Max}(x_1, x_2) = \begin{cases} x_2, & x_1 \leq x_2 \\ x_1, & x_1 > x_2 \end{cases}$$

○ (II)

$$\text{Min}(x_1, x_2) = \begin{cases} x_1, & x_1 \leq x_2 \\ x_2, & x_1 > x_2 \end{cases}$$

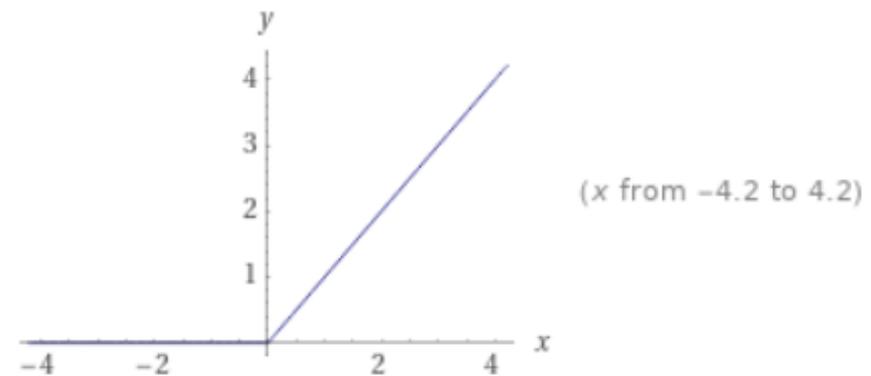
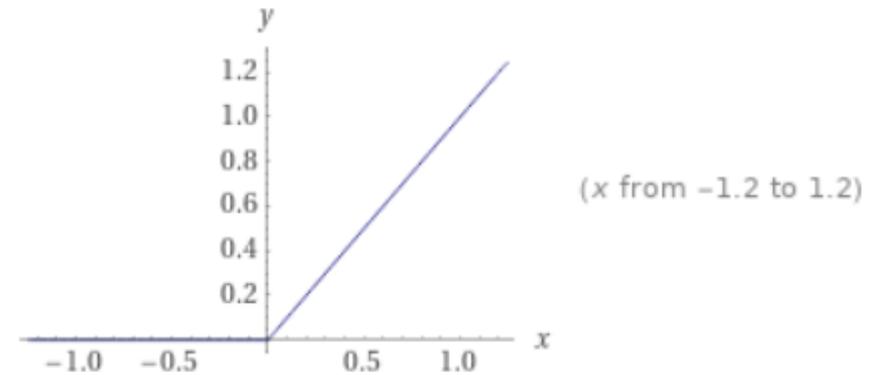
Microsoft Excel Solver 基本操作簡介

Source: <https://www.wolframalpha.com/input?i=max%280%2C+x%29>

$$\text{Max}(x_1, x_2) = \begin{cases} x_2, & x_1 \leq x_2 \\ x_1, & x_1 > x_2 \end{cases}$$

(I) 例子:

$$\text{Max}(x, 0) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$



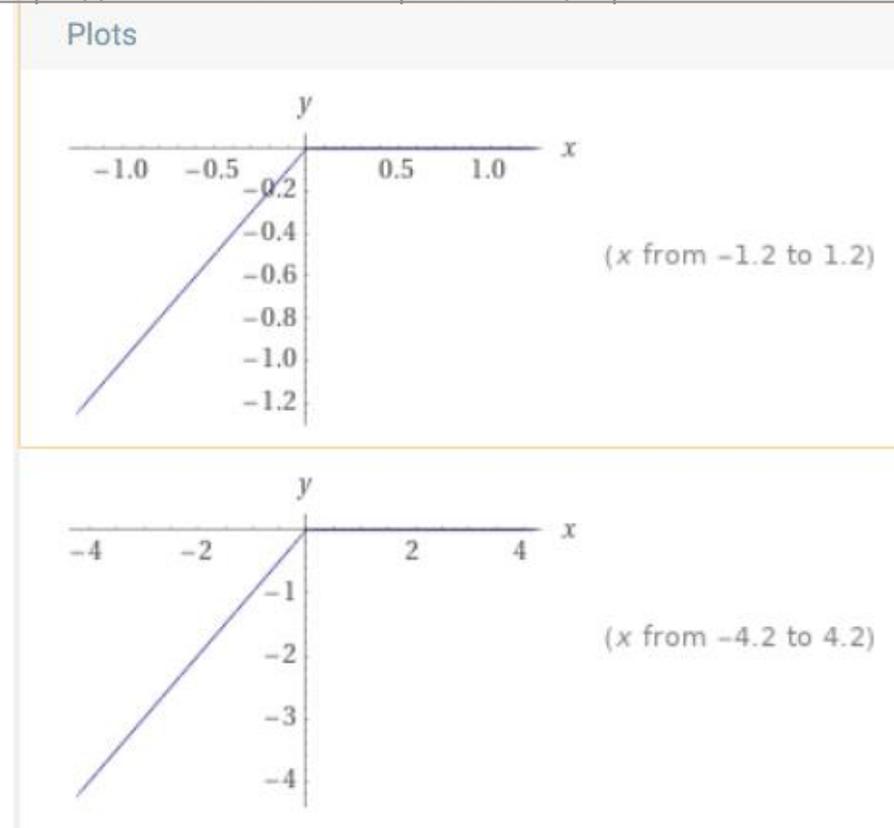
Microsoft Excel Solver 基本操作簡介

Source: <https://www.wolframalpha.com/input?i=min%280%2C+x%29>

$$\text{Min}(x_1, x_2) = \begin{cases} x_1, & x_1 \leq x_2 \\ x_2, & x_1 > x_2 \end{cases}$$

II. 例子:

$$\text{Min}(x, 0) = \begin{cases} x, & x \leq 0 \\ 0, & x > 0 \end{cases}$$

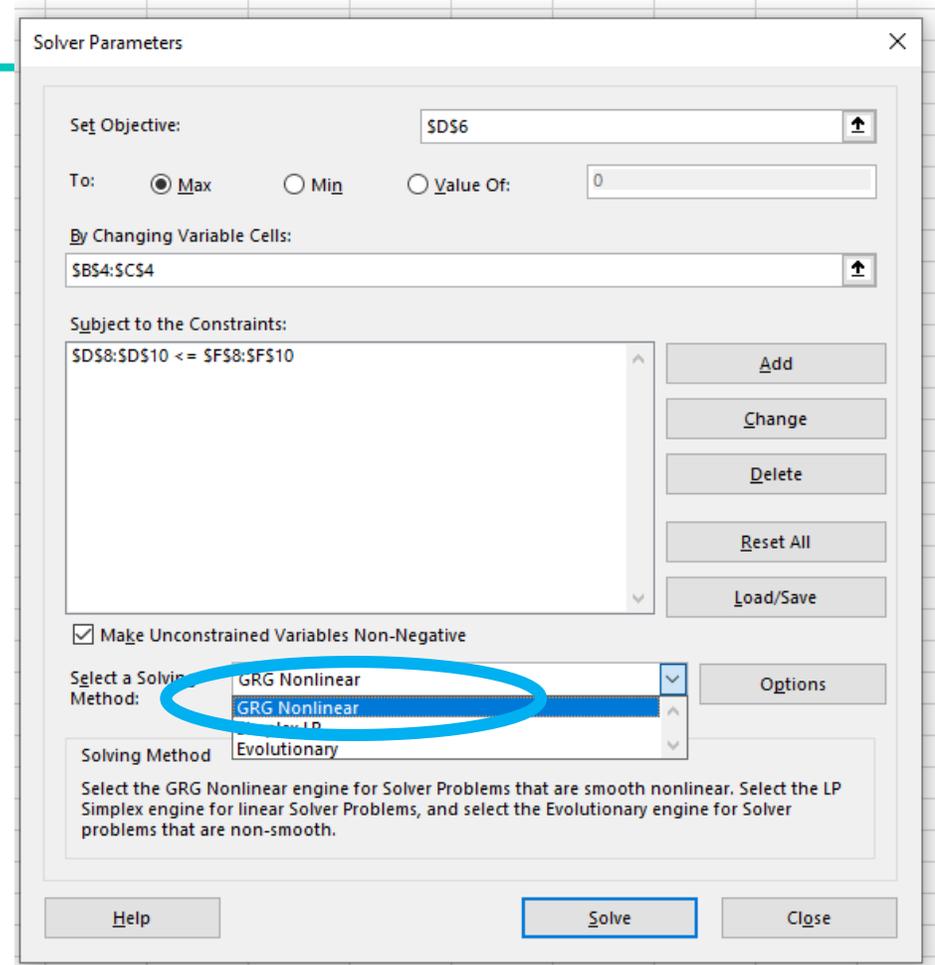


Microsoft Excel Solver 基本操作簡介

○ 例子：

$$\begin{aligned} & \text{maximize}_{x,y} && x + \min\{2, y\} \\ & \text{subject to} && 1.5x + y \leq 9 \\ & && 150x + 70y \leq 750 \\ & && 0 \leq y \leq 6, 0 \leq x \end{aligned}$$

○ 解決以上問題時，我們選擇“GRG Nonlinear”求解。



Microsoft Excel的應用-線性迴歸

- 假設一家公司有七名員工。下表列出了上年度員工請假天數的數據資料。

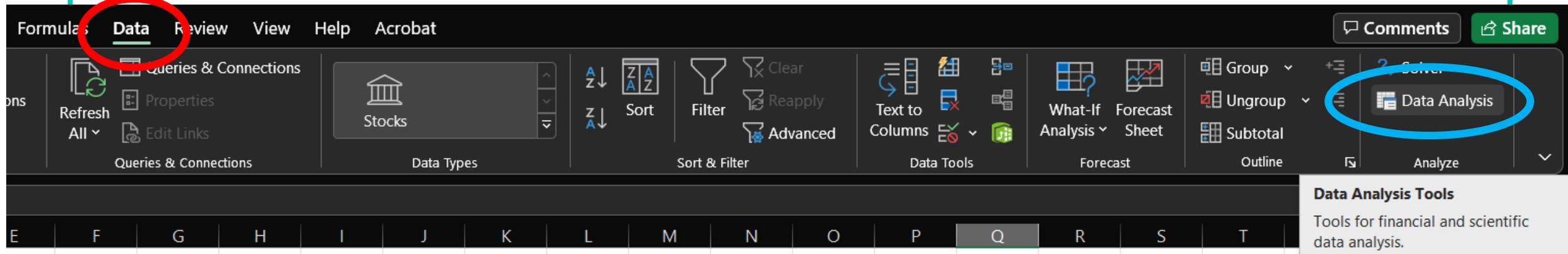
- X: 員工的年資
- Y: 上年度請假天數

X	Y
2	8
5	7
7	5
3	12
8	3
3	9
7	5

假如我們希望使用線性迴歸分析員工年資 (X) 和請假天數 (Y) 的線性關係。

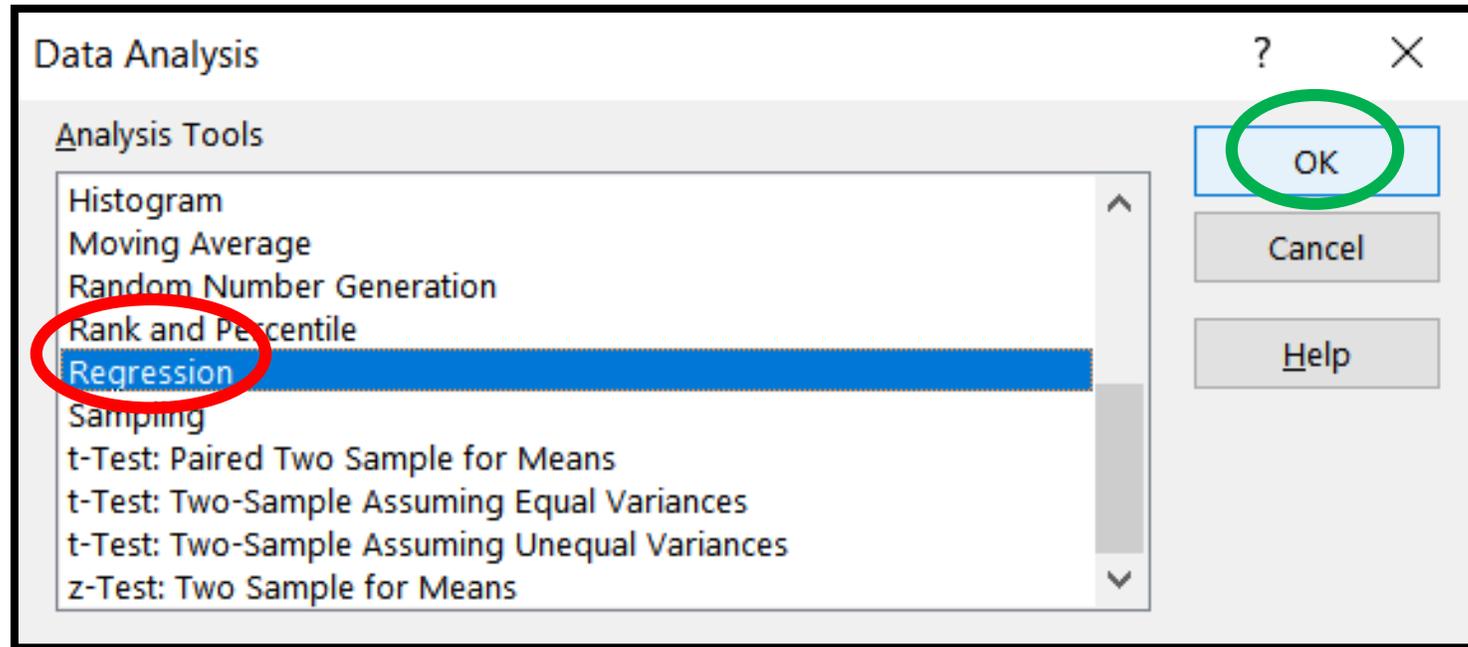
Microsoft Excel的應用-線性迴歸

- 在“Data”選取“Data Analysis”。接著會彈出一個新的視窗。



Microsoft Excel的應用-線性迴歸

- 在新視窗中選取“Regression” 並按 “OK” 。



Microsoft Excel的應用-線性迴歸

○ 我們可以按圖中示範，在不同位置輸入各項資料及設定。

○ 例如：

○ Y 變量的範圍是 \$B\$1:\$B\$8

○ X 變量的範圍是 \$A\$1:\$A\$8

○ 當設定完畢，按“OK”。

The screenshot displays an Excel spreadsheet with data in columns A and B, and the Regression dialog box open. The data table is as follows:

	A	B
1	X	Y
2	2	8
3	5	7
4	7	5
5	3	12
6	8	3
7	3	9
8	7	5

The Regression dialog box is configured with the following settings:

- Input Y Range: \$B\$1:\$B\$8
- Input X Range: \$A\$1:\$A\$8
- Labels
- Constant is Zero
- Confidence Level: 90 %
- Output options:
 - Output Range: \$A\$10
 - New Worksheet Ply:
 - New Workbook
- Residuals:
 - Residuals
 - Residual Plots
 - Standardized Residuals
 - Line Fit Plots
- Normal Probability:
 - Normal Probability Plots

Buttons: OK, Cancel, Help

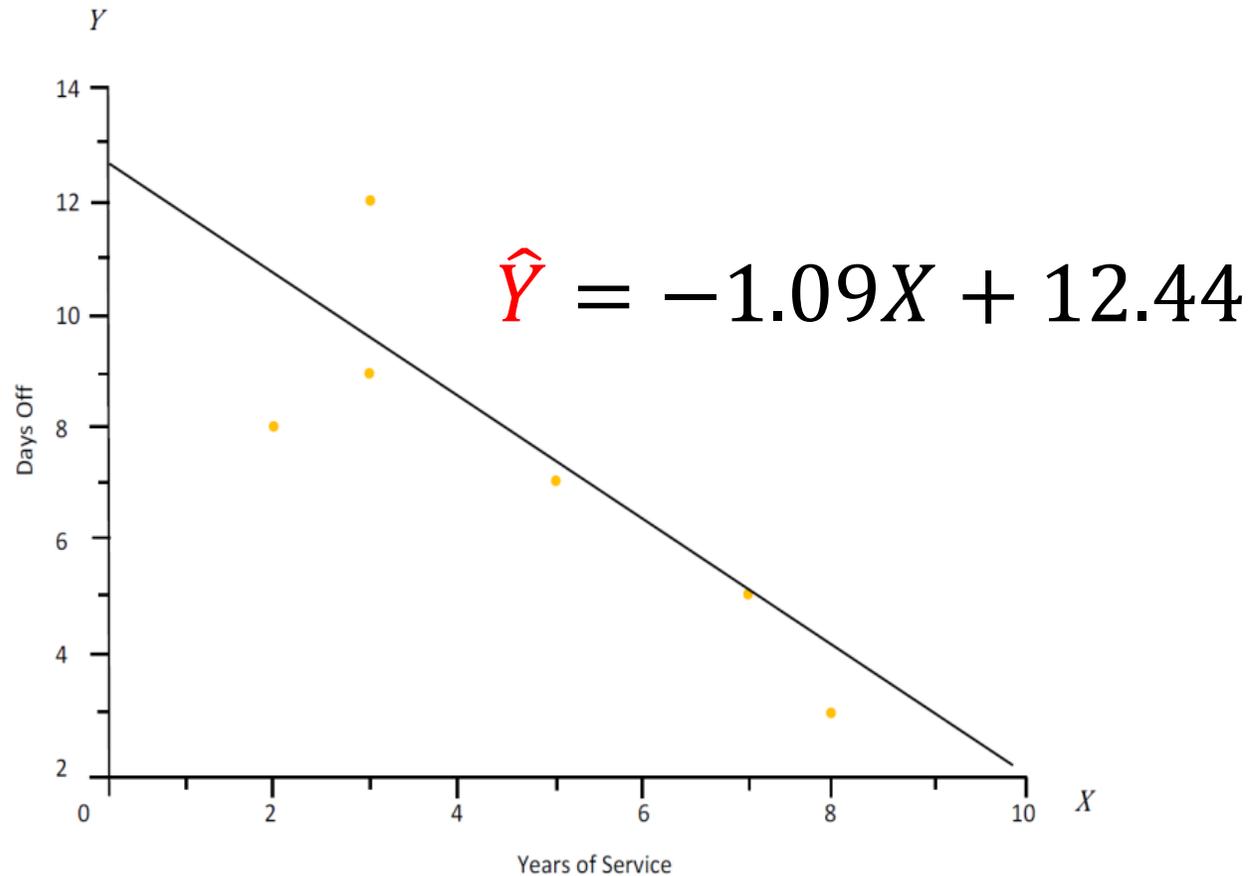
Microsoft Excel的應用-線性迴歸

Y軸截距
(Y-intercept)

斜率
(Slope)

SUMMARY OUTPUT									
<i>Regression Statistics</i>									
Multiple R	0.863506052								
R Square	0.745642702								
Adjusted R	0.694771242								
Standard E	1.65742536								
Observatio	7								
<i>ANOVA</i>									
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	1	40.26470588	40.26470588	14.65738758	0.012266446				
Residual	5	13.73529412	2.747058824						
Total	6	54							
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 90.0%</i>	<i>Upper 90.0%</i>	
Intercept	12.44117647	1.553168751	8.01018979	0.000489973	8.448629092	16.43372385	9.311466306	15.5708866	
X	-1.088235294	0.284246104	-3.828496778	0.012266446	-1.818913166	-0.357557422	-1.661004944	-0.51546564	

Microsoft Excel的應用-線性迴歸



總結

- Microsoft Excel Solver 基本操作簡介
 - 基本操作簡介
 - 設定整數變量或者二進制變量
 - 特別數學函數介紹: Max 和 Min
- Microsoft Excel的應用-線性迴歸

參考資料及聯絡方法

- <https://cosmo.se.cuhk.edu.hk/cosmo2024>
 - cosmo@se.cuhk.edu.hk
- <https://www.se.cuhk.edu.hk/>